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RISK CONTROL

Note

MDB Sovereign Loan Credit Performance and PCT: Public Data Analysis

1. Introduction

The business model of Multilateral Development Banks (MDBs) involves borrowing low-cost funds in the bond market and lending at low interest rates to Member Country (MC) Sovereigns Obligors (SOs) and Non-Sovereign Obligors (NSOs). This business model relies heavily on the Preferred Creditor Treatment (PCT) that MDBs enjoy.

PCT denotes the de facto preferential treatment that financially distressed sovereigns give to multilateral institutions including MDBs, Specialised Multilateral Insurers (SMIs) and the International Monetary Fund (IMF). Without this being contractually required defaulting sovereigns try to avoid impairing the debt of multilaterals and, when they fall into arrears, attempt, in restructurings, to repay multilateral debt in full even if other debt claims are written down.

PCT is widely discussed and many MDB activities would be scarcely possible without it. However, there have been few attempts to quantify PCT. This note uses public data on sovereign debt

(i) to quantify the credit performance of four prominent MDBs and
(ii) to analyse the magnitude of the PCT they enjoy.

The MDBs we examine are the International Bank for Reconstruction and Development (IBRD), the African Development Bank (AfDB), the Interamerican Development Bank (IDA) and the Asian Development Bank (ADB). These four institutions are prominent sovereign focussed MDBs in that loans to NSOs make up small fractions of their portfolios. Some major MDBs that we do not consider here lend primarily to non-sovereign obligors.

On (i), we proceed as follows:

- We collect three decades of sovereign exposure and arrears data from the annual financial statements of the banks included.
- Using this information, we compute the 1-year probability of default (PD) for sovereigns that at the start of the year are not in arrears.
- We perform similar calculations of PDs for sovereign borrowers broken down by geographical region, per capital GDP and the first versus second half of the sample period.
- We also compute the 1-year probability of emergence (PE) from default for sovereigns that start the year in arrears.
- Under the assumption that MDBs are fully repaid interest and principal when sovereign borrowers emerge from default, the economic cost of a sovereign default for the MDB is the loss interest on the interest.
- Using our estimates of PE and assuming a particular value for the interest rate, we can, under these assumptions, compute the Loss Given Default (LGD) or the percentage provision that a bank should make when a default occurs.

On (ii), we do the following:

- We compute the magnitude of PCT by combining the MDB exposure and default data with comparable information of Foreign Currency (FC) bank loans and FC bonds.
- We align the sample precisely in the sense that we consider only years and countries for which the sovereigns in question have outstanding debt *both* to the MDB we are examining *and* to either: (a) FC bank loans or (b) FC bonds.
- Then, we compute the sample PDs for MDB loans with those for FC loans and FC bonds.
- The difference between the PDs reveals the magnitude of the favourable treatment that MDBs receive from sovereign borrowers compared to private sector lenders.

We implement our approach for individual MDBs and then provide averages across the MDBs that have exposure to the sovereign loan category in question. In some cases, especially when we consider a sub-sample such as low-income countries, very few default observations lie behind the PD and LGD estimates we report. The results must be treated with caution. We compute Monte Carol based estimates of standard errors for PDs. These are obtained by simulating the dataset employed, generating PD estimates for each Monte Carlo replication, and then computing the standard errors of the simulated PD estimates. This approach permits the reader to judge the statistical reliability of the findings.

The document is organised as follows. Section 2 presents the estimates of sovereign loan PDs and analyses sovereign loan PDs conditioning on a sample in which the sovereigns involved have also borrowed from Foreign Currency (FC) loan or bond markets. The PDs on MDB loans are then compared to those of FC loans and bonds to obtain an estimate of the magnitude of PCT. Section 3 presents estimates of MDB LGDs using analytical expressions that depend on interest rates and the Probability of Emerging from default (PE). Section 4 concludes.

2. Estimation of PDs on MDB Loan and FC Loan/Bond

Analysis

The analysis of MDB credit performance that we carry out may be summarised as follows.

- We calculate probabilities of default (PDs) and probabilities of emergence from default (PEs) for the MDB's sovereign loans using data collected from their historical published accounts and financial statements.
- We estimate standard errors for the estimates using Monte Carlo methods. The estimates are based on data from the period 1991 to 2020 (30 years in total). This is for all the development banks except for AfDB, for which data from 1997 to 2020 are employed. The detailed methodology is given in Appendix 3.
- Starting from estimates of PEs, we compute LGD rates using the fact that the economic loss experienced by MDBs on defaulted loans is the interest on the interest. The methodology is explained in Appendix 2.2 and the results are presented in Section 3.

We compute 'MDB average' outcomes from the results for the four individual MDBs we study. By 'average' we mean simple arithmetic averages (not, for example, exposure-weighted) of the individual-MDB results.[1]

The approach in the third bullet point is made possible because MDBs almost never write-off delinquent sovereign loans. Instead, they wait until the sovereign exits its non-accrual status. Typically, this occurs when the sovereign itself or a donor repays the overdue MDB interest and principal. In this case, the MDB loses no more than the foregone interest on the interest. Thus, it is possible, from (a) the average length of time that one may expect the sovereign to remain in default and (b) the level of interest rates, to calculate the expected foregone interest, i.e., the expected Loss Given Default on the loan.[2]

The analysis of PCT that we perform may be summarised as follows.

[1] Where fewer than four have exposure to a particular category of exposures (which is most obviously true for geographical regions), we average only over those MDBs that have non-zero exposure to the category in question at some point in the sample period.

[2] To be specific, the LGD under these assumptions depends on (a) from the estimates of PD and PE, (b) the level of interest rates, and (c) the assumption that default and re-emergence follows a simple Markov chain.

- We combine the information on MDB sovereign lending with data on sovereign FC loan and bond borrowing and arrears from the World Bank debt tables and the Bank of Canada-Bank of England dataset.
- We calculate MDB PDs for countries and years in which the sovereigns in question have also borrowed either from the international bond market or through loans from bank.
- For the subsample of such observations, we directly compare the performance of debt held by MDBs and that held by private sector investors in FC loans and bonds.

Results

As already stated, the four MDBs we study are ADB, AfDB, IaDB and IBRD. Results for the individual banks are shown in Appendix 1. We calculate PDs and PEs based on sub-samples corresponding to the following categories of observations:

1. **Regions:** East Asia & Pacific, Europe & Central Asia, Latin America & Caribbean, Middle East & North Africa, Sub-Saharan Africa, and South Asia.
2. **Income Groups:** Low income, Lower middle income, Upper middle income, and High income.
3. **Periods:** Before 2005 (including 2005) and After 2005.

These classifications appear to us the most obvious way to split the sample. Another possibility would be to divide data according to the initial rating of the country involved. Estimating PDs conditional on rating is a further step which, while worthwhile, we postpone until a later study. Conditioning on rating will not be necessary for our comparison of MDB to private sector debt performance since we shall precisely match samples by considering years/country observations for which both MDB and private loans/bonds exist.

Regions and Income Groups are based on classifications employed by the World Bank. The above estimations employ either the full sample or the sub-samples as defined above. We calculate Monte Carlo estimates of the standard deviations for each PD and PE as described in Appendix 2. The Monte Carlo estimates are based on 10,000 replications of the full dataset

Table 2.1 shows the averages of four MDBs. To reiterate, the outcomes shown are the averages of individual MDB results for banks that have non-zero exposure for the category in question. For example, the average PD for East South Asia & Pacific according to observations from the public data is calculated as (0.36%+0%)/2=0.18%, where the values 0.36% and 0% come from the relevant individual MDB tables in Appendix 1.

Table 2.1: Average Default Probabilities for Four MDBs

MDB Average	Whole sample		Sample with FC bank loans exposure				Sample with FC bonds exposure			
	MDBs		MDBs		Loans		MDBs		Bonds	
	PD	StD	PD	StD	PD	StD	PD	StD	PD	StD
MDB level	0.67	0.28	0.48	0.30	1.55	0.83	1.87	1.07	6.69	2.61
Americas & Caribbean	0.14	0.08	0.12	0.12	1.00	0.57	0.17	0.17	5.64	1.52
East South Asia & Pacific	0.27	0.16	0.00	0.00	3.29	1.88	0.00	0.00	4.00	3.59
Europe & Central Asia	0.07	0.08	0.48	0.49	0.51	0.54	0.00	0.00	5.66	2.33
Middle East & North Africa	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.92	1.71
Sub-Saharan Africa	1.68	0.61	1.69	1.02	1.86	1.17	6.67	3.61	11.98	4.51
Low income	1.73	1.20	3.85	3.53	0.00	0.00	0.00	0.00	0.00	0.00
Lower middle income	0.45	0.27	0.48	0.35	1.56	0.86	2.13	1.17	9.85	3.86
Upper middle income	0.45	0.44	0.83	0.80	1.32	1.08	0.00	0.00	3.51	1.30
High income	2.31	2.13	1.85	1.63	6.55	5.94	1.85	1.63	8.01	6.16
1991-2005	1.30	0.59	1.04	0.58	3.46	1.89	5.01	2.52	8.12	4.69
2006-2020	0.20	0.19	0.11	0.11	0.22	0.23	0.15	0.16	5.74	2.21

Note: Outcomes broken down by region, income group and year group (Data for AfDB from 1997). StD denotes standard deviation. All the results are reported in percentages.

We can summarise our findings as follows:

1. 1-year Sovereign PDs using the public data are 0.67%.
2. PDs vary from 1.68% for sub-Saharan Africa to very low probabilities of 27 bps or lower for all other regions.

3. Low-income country PDs are relatively high at 1.73%. High-income countries show a PD of 2.31%. This is noticeably higher than the PDs for Middle income countries. The standard error for High income countries is very high, however, reflecting the fact that there are very few such observations in the sample and suggesting the non-monotonicity in income level is not statistically significant.
4. PDs have fallen markedly over time from 1.30% to 0.20%.
5. The precise comparison between PDs for MDBs and PDs for FC loans and bonds suggests that PDs for bonds are 6.69% while for a matched sample of MDB data we obtain 1.87%, a ratio of about 3.5. We find the same factor for FC bank loans versus MDB loans.
6. The only result where MDBs have higher PD than the FC bank loans is for the sub-sample associated with low-income countries. This finding is not significant for the following reason. Only 1 low-income country in the MDB data, Sudan, defaulted (this occurred in 1994). According to the bank loan data, Sudan defaulted in 1991, but this year is before the start of our sample and, hence, does not contribute to the PD estimate.

Table 2.2 shows the Weighted Average Borrower Rating (WABR) of the MDBs, calculated by Moody's for the fiscal year 2020. These WABRs are strikingly low compared to what one observes in commercial banks in developed countries. To illustrate, Basel adopted 100% risk weights for loans in the vicinity of Ba1 in part because regulators believed that the credit quality of typical US bank loan portfolios was in that range.

Table 2.2 also presents PDs corresponding to those ratings extracted from the S&P 2021 annual default study. We smoothed the PDs with interpolation to ensure the PDs are monotonic as the credit quality decreases. One may note that these PDs are comparable to the ones for loans and bonds. However, it should be emphasised that the PDs in Table 2.2 do not allow for PCT, so they are not comparable to the MDB's actual PDs. The table demonstrates how misleading it is to apply rating agency historical PDs to MDB portfolios.

Table 2.2: Moody's WABR

MDB	WABR	PD
AfDB	B2	2.41%
ADB	Ba3	1.06%
IBRD	B1	1.74%
IaDB	B3	8.11%

Source: Moody's (2021) and S&P (2022).

3. LGD Results

Table 3.1 displays the results of our calculations of LGDs. These follow directly from the Probability of emerging from default and the assumed level of the interest rate. The methodology for computing LGDs in this manner is explained in Appendix 3.

Table 3.1: Average LGD for MDBs

		LGD	
	PE	r=2.5%	r=5%
MDB average	23.46%	0.71%	2.36%
ADB	23.08%	0.73%	2.44%
AfDB	8.15%	5.06%	13.28%
IaDB	50.00%	0.11%	0.41%
IBRD	12.63%	2.38%	7.03%

The average PE or probability of emerging from default over a 1-year horizon is 23.46%. Sovereign LGDs for MDBs are 0.71% or 2.36% depending on whether one assumes an interest rate of 2.5% or 5%. The results vary across MDBs depending on the 1-year probability that sovereigns in arrears will exit the arrears state. These figures may be compared with the estimates of LGDs experienced by private sector holders of sovereign debt. Moody's (2017) and Trebesch and Cruces (2013) provide estimates of mean sovereign LGDs equalling 35% and 45%, respectively. The review of the papers is given in Appendix 3. Clearly, the mean LGD rates in the above cited studies are much higher than those estimated based on PEs. This presumably reflects the fact that MLI defaulted loan recoveries benefit from PCT.

4. Conclusion

This document sets out the results of a statistical investigation of 1-year Probabilities of Default (PDs) and mean Loss Given Default (LGD) rates for sovereign loans made by Multilateral Development Banks (MDBs). Data for four prominent, sovereign focussed MDBs (namely ADB, AfDB, IBRD, IDB) are considered. The data are collected from the annual financial statements of these institutions and, hence, are in the public domain.

We assume that default events and emergence from default follow a simple-two-state Markov chain in which the two states consist of being in arrears/default or not. We are then able to compute the sample PDs and Probabilities of Emergence (PE) from default. From the estimated PEs, an assumed level of interest rates and the supposition that, when sovereigns emerge from default, MDBs receive full repayment of forgone principal and interest but not the interest on the interest, we are able to compute the LGDs experienced by MDBs. We report PD and LGD estimates by MDB and averaged across the four MDBs considered.

Our main findings are as follows.

- The 1-year sovereign PD averaged across the four MDBs is 0.67%. This is a strikingly low number given that the weighted average ratings of the MDBs in question is single B for 3 banks and low double B for the fourth.
- PD estimates range from 1.68% for sub-Saharan Africa to values of 27 bps or less for other regions.
- Low-income-country PDs are relatively high at 1.73%. Estimated PDs appear not monotonic in income level. This result is not statistically dependable, however, since the number of observations in the sample for High income countries is very few.
- PDs have fallen very substantially over time from an average of 1.30% in the first half of the sample period to 0.20% in the second half.
- Our investigation of PCT suggests that the sovereign PDs of MDB loans are a factor of 3.5 times lower than those experienced by the FC loan or FC bond markets.
- MDB LGDs computed under our assumptions that the economic cost to the MDB is the interest on the interest are around 5 to 10% depending on the estimated PE (which varies across banks) and the level of interest rates. These figures are much lower than the estimates in the literature of LGDs experienced by private sector lenders to sovereigns which range from 35% to 45%.

If PDs are 3.5 times lower for MDBs (compared to private lenders) and LGDs are about 4 times lower, then 1-year Expected Credit Losses (ECLs), which equal the product of PDs and LGDs, are 14 times lower for MDBs compared to those of private sector lenders.

The proportional effect on required capital is more complex to compute. Calculations we have performed elsewhere for MDB portfolios using industry-standard Credit Portfolio Models (CPMs) with and without 'PCT adjustments' to PDs and LGDs suggest that capital is around 6 times lower if one allows for PCT.

Some MDBs already adjust their PDs and LGDs for PCT so this does not mean that the capital that MDBs need to hold is 6 times lower than what they currently compute. However, capital multipliers of 6 are much larger than the allowance that rating agencies make in their assessments of capital adequacy when rating MDBs compared to commercial banks.

References

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Trebesch, J. and C. Cruces (2013) "Sovereign Defaults: The Price of Haircuts," mimeo.

Appendix 1: Individual MDB PD/PE Results

A1.1 PD/PE Estimates for each MDB

Outcomes are broken down by region, income group and year group (Data for AfDB from 1997). StD stands for the standard deviation. All the results are reported in percentages.

Table A1.1: Default Probabilities with Monte Carlo Estimations for ADB

ADB	Whole sample		Sample with FC bank loans exposure				Sample with FC bonds exposure			
	MDB		MDB		Loans		MDB		Bonds	
	PD	StD	PD	StD	PD	StD	PD	StD	PD	StD
MDB level	0.45	0.27	0.00	0.00	3.09	1.76	0.00	0.00	4.00	3.59
Americas & Caribbean	0.00	0.00								
East South Asia & Pacific	0.55	0.32	0.00	0.00	3.09	1.76	0.00	0.00	4.00	3.59
Europe & Central Asia	0.00	0.00								
Middle East & North Africa	0.00	0.00								
Low income	0.00	0.00								
Lower middle income	0.26	0.25	0.00	0.00	3.09	1.76	0.00	0.00	4.00	3.59
Upper middle income	0.45	0.46								
High income	2.56	2.34								
1991-2005	0.88	0.64	0.00	0.00	7.32	3.93	0.00	0.00	10.00	8.97
2006-2020	0.23	0.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table A1.2: Default Probabilities with Monte Carlo Estimations for AfDB

AfDB	Whole sample		Sample with FC bank loans exposure				Sample with FC bonds exposure			
	MDBs		MDBs		Loans		MDBs		Bonds	
	PD	StD	PD	StD	PD	StD	PD	StD	PD	StD
MDB level	1.49	0.50	1.22	0.71	0.84	0.61	6.52	3.58	10.91	4.19
Middle East & North Africa	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Sub-Saharan Africa	1.72	0.56	1.42	0.84	0.99	0.72	6.52	3.58	10.91	4.19
Low income	2.96	1.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Lower middle income	0.93	0.53	1.21	0.90	0.61	0.65	6.67	3.57	12.77	4.74
Upper middle income	0.78	0.85	2.33	2.20	2.56	2.34				
High income	5.88	5.32								
1991-2005	3.14	1.10	3.23	1.86	1.20	1.21	18.75	9.30	12.00	6.15
2006-2020	0.29	0.28	0.00	0.00	0.65	0.69	0.00	0.00	10.00	4.93

Table A1.3: Default Probabilities with Monte Carlo Estimations for IaDB

IaDB	Whole sample		Sample with FC bank loans exposure				Sample with FC bonds exposure			
	MDBs		MDBs		Loans		MDBs		Bonds	
	PD	StD	PD	StD	PD	StD	PD	StD	PD	StD
MDB level	0.42	0.25	0.25	0.24	0.89	0.51	0.35	0.35	5.58	1.51
Americas & Caribbean	0.42	0.25	0.25	0.24	0.89	0.51	0.35	0.35	5.58	1.51
Lower middle income	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.42	4.29
Upper middle income	0.50	0.35	0.00	0.00	0.76	0.54	0.00	0.00	3.73	1.53
High income	0.58	0.64	3.70	3.26	4.00	3.59	3.70	3.26	8.33	5.26
1991-2005	0.60	0.42	0.00	0.00	2.36	1.43	0.00	0.00	3.74	1.91
2006-2020	0.26	0.26	0.44	0.45	0.00	0.00	0.61	0.65	7.14	2.35

Table A1.4: Default Probabilities with Monte Carlo Estimations for IBRD

IBRD	Whole sample		Sample with FC bank loans exposure				Sample with FC bonds exposure			
	MDBs		MDBs		Loans		MDBs		Bonds	
	PD	StD	PD	StD	PD	StD	PD	StD	PD	StD
MDB level	0.31	0.12	0.46	0.23	1.35	0.43	0.61	0.35	6.26	1.16
Americas & Caribbean	0.00	0.00	0.00	0.00	1.11	0.64	0.00	0.00	5.70	1.53
East South Asia & Pacific	0.00	0.00	0.00	0.00	3.49	2.00	0.00	0.00	4.00	3.59
Europe & Central Asia	0.15	0.15	0.48	0.49	0.51	0.54	0.00	0.00	5.66	2.33
Middle East & North Africa	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.85	3.42
Sub-Saharan Africa	1.63	0.65	1.97	1.20	2.73	1.62	6.82	3.64	13.04	4.83
Low income	2.22	2.07	7.69	7.07	0.00	0.00			0.00	0.00
Lower middle income	0.59	0.31	0.70	0.50	2.52	1.03	1.85	1.10	12.23	2.83
Upper middle income	0.09	0.10	0.18	0.18	0.62	0.36	0.00	0.00	3.28	1.07
High income	0.20	0.20	0.00	0.00	9.09	8.28	0.00	0.00	7.69	7.07
1991-2005	0.59	0.22	0.95	0.47	2.96	0.97	1.29	0.77	6.73	1.75
2006-2020	0.00	0.00	0.00	0.00	0.23	0.23	0.00	0.00	5.83	1.56

Table A1.2 shows the results for AfDB. We check that why AfDB has higher PD than bank loans.

- There are 3 defaults from non-default in AfDB data: Côte d'Ivoire in 2000 and 2003, Gabon in 1998.
- There are 2 defaults from non-default in bank loans: Gabon in 1999 and Cameroon in 2019.
- Côte d'Ivoire starts as already in default in 1997 in the bank loan data so it is not counted in the PD calculation. It emerges back in 1999 and does not default again whereas it defaults in 2000 and 2003 in AfDB data and emerges back in 2009. So, this is an example where one defaults in MDB data and does not default in bank loans.
- There are other countries (Ethiopia, Madagascar and Congo Rep.) that are in default in bank loans and are not in default in AfDB, which is in favour of AfDB. But these are not counted in PD calculations because the year before the default is not in the overlapped data.

Table A1.3 shows the results for IaDB. IaDB has slightly higher PD (0.44%) than loans (0%) for 2006-2020. There is 1 default (Venezuela in 2018) for IaDB and 0 for loans. This is another example where one defaults in MDB data and does not default in bank loans. For the 2006-2020 period, Argentina (always) and Nicaragua (until 2009) are in default in loans, but they are not counted in the PD since they became in default before 2006. These countries are not in default for IaDB, which is in favour of the MDB.

Table A1.4 shows the results for IBRD. MDB has higher PD for low-income countries than loans. Only 1 low-income country, Sudan, defaulted in 1994 in the IBRD data whereas Sudan starts with already in default in 1991 in the bank loan data. So, it is not counted in the PD calculation.

Table A1.5 shows the countries that are in default for the MDBs and not for FC bank loans or bonds and their corresponding years. There are common countries across MDBs such as Côte d'Ivoire, Democratic Republic of the Congo, Nicaragua and Zimbabwe.

Table A1.5: Countries in Default for MDB but not for FC Bank Loans or Bonds

	FC Bank Loans	FC Bonds
ADB	-	-
AfDB	Côte d'Ivoire: 2000-2001 & 2003-2008 Gabon: 1998	Dem. Rep. of the Congo: 1997-2003 Zimbabwe: 2000-2008
IaDB	Venezuela: 2018-2020	Nicaragua: 1991
IBRD	Bosnia and Herzegovina: 2000-2001 Côte d'Ivoire: 2001 & 2005-2007 Guatemala: 1991-1992	Dem. Rep. of the Congo: 1997-2002 Guatemala: 1991-1992 Nicaragua: 1991 Zimbabwe: 2001-2008

A1.2 Default events for each MDB

ADB default events: Myanmar (1998), Nauru (2001), Marshall Islands (2006). Note that default by the Marshall Islands is not identified in the ADB's annual reports but we have been informed that this event occurred. Also, judging by the annual reports, the default of Nauru ended in 2007. However, we understand that it continued until the end of 2007 and, hence, we have adjusted the data used in the calculations accordingly.

AfDB default events: Burundi (2000, 2004), Central African Republic (2006), Côte d'Ivoire (2000, 2003), Gabon (1998), Guinea (1998), Seychelles (2000), Zimbabwe (2000).

IaDB default events: Suriname (1992, 2001), Venezuela (2018).

IBRD default events: Bosnia and Herzegovina (2000), Republic of the Congo (1998), Côte d'Ivoire (2001, 2005), Seychelles (2003), Sudan (1994), Zimbabwe (2001).

Appendix 2: Standard Deviation Calculation

After estimating a PD, we can calculate its standard deviation using Monte Carlo simulation. The process is described below:

1. Calculate the cut-off point C using the estimated PD:

$$C = \Phi^{-1}(PD) \tag{A2.10}$$

2. Simulate latent variables:

$$X_i = \sqrt{\rho} \times f_i + \sqrt{1-\rho} \times \varepsilon_i \tag{A2.11}$$

   where f_i and ε_i are i.i.d. standard normal, $i = 1,2,\dots,n$. The correlation parameter ρ is calculated as follows:

$$\rho = 0.12\frac{1-e^{-50\times PD}}{1-e^{-50}} + 0.24\left(1 - \frac{1-e^{-50\times PD}}{1-e^{-50}}\right) \tag{A2.12}$$

3. Generate default/non-default indicators for each i:

$$I_i = \begin{cases} 1, \text{if } X_i < C \\ 0, \text{otherwise} \end{cases} \tag{A2.13}$$

4. Calculate the simulated PD:

$$PD_s = \frac{\sum_{i=1}^{n} I_i}{n} \tag{A2.14}$$

5. Repeat step 2-4 N times to obtain an $N \times 1$ array of PDs.

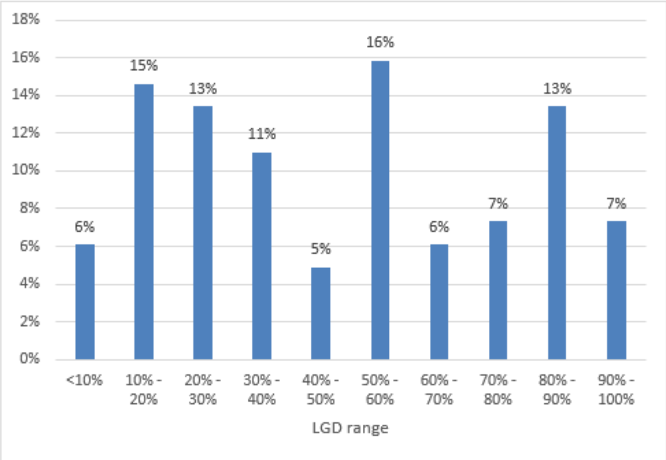
6. Calculate the standard deviation of the array in step 5.

Appendix 3: Estimation of Sovereign LGDs

A3.1 Literature Review

Trebesch and Cruces (2013) construct an extensive dataset of 187 distressed sovereign debt restructurings with external private-sector creditors, including both banks and bondholders. The restructurings occurred between 1970 and 2013 and were mainly in Emerging Markets. After aggregation at the level of sovereign counterparty, the dataset is reduced to 82 country-level observations.

Figure A3.1: Distribution of LGDs (SZ haircut)



Note: This figure shows the country level LGDs fall in each LGD value band. This figure is produced based on the T&C data.

Trebesch and Cruces present LGD estimates based on two different methodologies: (1) haircut SZ and (2) haircut market. Trebesch and Cruces argue in favour of using the SZ approach. Table A3.1 presents statistics of the Trebesch and Cruces LGD data.

Table A3.1: LGDs by SZ haircut approach

Measure	Haircut (SZ)
Min	3%
1st quartile	25%
Mean	48%
Median	50%
3rd quartile	74%
Max	97%
Standard deviation	28%

Source: T&C data. There are 82 country level LGDs employed in calculation.

Moody's (2017) employs data on defaulted sovereign bonds in the period 1998 to 2016. The dataset contains information on 23 defaults in total and contains estimates based on two approaches: (i) trading prices and (ii) cash flows. Table A2.2 shows statistics of LGDs estimated using the Moody's cash flows approach data.

Table A3.2: Statistics of Moody's cash-flow recovery rates

Measure	LGD
Min	3%
1st quartile	14%
Mean	35%
Median	38%
3rd quartile	50%
Max	71%
Standard deviation	23%

Note: 16 sovereign defaults observations are employed in the calculations.
Source: Moody's recovery rates data.

A3.2 LGD Calculation Methodology

The economic losses for an MDB in the event of arrears on a sovereign loan equal the discounted cost of the foregone "interest on the interest". The reason is that when a sovereign emerges from a state of default, it makes good unpaid interest on the principal, but does not typically pay interest on the unpaid interest.

The point is made in Figure A3.2 which sets out the expected cost to an MDB of a sovereign loan default assuming a par value of unity, a coupon rate of c and a constant interest rate of r. The array of terms in the figure shows the losses associated with unpaid interest in each year (reading across the columns) assuming different re-emergence from default years (corresponding to particular rows).

Figure A3.2: Losses on a MDB Loan in Arrears for Different Re-emergence Years

	Calendar Year					
Emergence year	Year 2	Year 3	Year 4	…	Year n+1	…
Year 2	$\frac{rc}{(1+r)^2}$	0	0	…	0	…
Year 3	$\frac{2rc}{(1+r)^3}$	$\frac{rc}{(1+r)^2}$	0	…	0	…
Year 4	$\frac{3rc}{(1+r)^4}$	$\frac{2rc}{(1+r)^3}$	$\frac{rc}{(1+r)^2}$	…	0	…
…	…	…	…	…	…	…
Year n+1	$\frac{nrc}{(1+r)^n}$	$\frac{(n-1)rc}{(1+r)^{n-1}}$	$\frac{(n-2)rc}{(1+r)^{n-2}}$	…	$\frac{rc}{(1+r)^2}$	…
…	…	…	…	…	…	…

Note: The figure shows the discounted value lost to an MDB when a sovereign loan falls into arrears. For simplicity, we look at a perpetual loan. Assume the contractual interest will be repaid when the sovereign re-emerges from its state of arrears. The economic loss consists of the lost 'interest on interest'.

The probability of re-emerging from arrears after i years is: $(1-P_E)^{i-1}P_E$. Under the simplifying assumption that the loan is perpetual, the expected Loss Given Default (LGD) may then be expressed as:

$$LGD = \sum_{i=2}^{\infty}(1-P_E)^{i-1}P_E\, rc \sum_{j=1}^{i-1}\frac{j}{(1+r)^{j+1}} = \frac{rc(1-P_E)}{(r+P_E)^2} \quad \text{(A3.1)}$$

The derivation required to calculate the term on the right-hand side of equation (A3.1) is provided below. As one may observe from equation (A3.1), the LGD depends on the interest rate (assumed constant) and the probability of emerging from arrears over a one-year horizon, P_E, (also assumed constant).

Let r and c be the interest and coupon rates respectively. Let p_E be the probability of re-emerging from arrears. Then, the LGD is given by,

$$X = \sum_{i=2}^{\infty}(1-p_E)^{i-1}p_E rc\left[\sum_{j=1}^{i-1}\frac{j}{(1+r)^{j+1}}\right] = \sum_{i=2}^{\infty} p_E rc X_i \quad \text{(A3.2)}$$

Here,

$$X_i = (1-p_E)^{i-1}\sum_{j=1}^{i-1}\frac{j}{(1+r)^{j+1}} \quad \text{(A3.3)}$$

Then for $i = 2,3,4$ we have,

$$X_2 = (1-p_E)\frac{1}{(1+r)^2} \quad \text{(A3.4)}$$

$$X_3 = (1-p_E)^2\left(\frac{1}{(1+r)^2}+\frac{2}{(1+r)^3}\right) \quad \text{(A3.5)}$$

$$X_4 = (1-p_E)^3\left(\frac{1}{(1+r)^2}+\frac{2}{(1+r)^3}+\frac{3}{(1+r)^4}\right) \quad \text{(A3.6)}$$

Thus for $i = n$,

$$X_n = (1-p_E)^{n-1}\left(\frac{1}{(1+r)^2}+\frac{2}{(1+r)^3}+\cdots+\frac{n-1}{(1+r)^n}\right) \quad \text{(A3.7)}$$

Then,

$$
\begin{aligned}
X_2 + X_3 + X_4 + \cdots = &\frac{(1-p_E)}{(1+r)^2}\left[1 + (1-p_E) + (1-p_E)^2 + \cdots + (1-p_E)^{n-1} + \cdots\right] \\
&+ \frac{2(1-p_E)^2}{(1+r)^3}\left[1 + (1-p_E) + (1-p_E)^2 + \cdots + (1-p_E)^{n-1} + \cdots\right] \\
&+ \frac{3(1-p_E)^3}{(1+r)^4}\left[1 + (1-p_E) + (1-p_E)^2 + \cdots + (1-p_E)^{n-1} + \cdots\right] \\
&+ \frac{n(1-p_E)^n}{(1+r)^{n+1}}\left[1 + (1-p_E) + (1-p_E)^2 + \cdots + (1-p_E)^{n-1} + \cdots\right] \qquad \text{(A3.8)} \\
&= \frac{1}{1-(1-p_E)} \sum_{i=1}^{\infty} \frac{i(1-p_E)^i}{(1+r)^{i+1}} \\
&= \frac{1}{p_E(1+r)} \sum_{i=1}^{\infty} i \left(\frac{(1-p_E)}{1+r}\right)^i \\
&= \frac{1-p_E}{p_E(r+p_E)^2}
\end{aligned}
$$

Thus, the LGD is given by:

$$
X = \sum_{i=2}^{\infty} p_E r c X_i = \frac{rc(1-p_E)}{(r+p_E)^2} \qquad \text{(A3.9)}
$$